BOUNDARY EFFECT ON CAVITATING FLOW PAST A CYLINDER

K. K. Shal'nev

Zhurnal Prikladnoi Mekhaniki i Teknicheskoi Fiziki, No. 3, pp. 103-108, 1965

The widespread method of reducing data obtained in model experiments on a flow of finite width to the conditions of an infinite flow has been verified experimentally. The experiments were conducted on cylinders from 5 to 20 mm in diameter at a relative constriction of the working chamber (d/b) varying from 0.05 to 0.40. The parameters characterizing the flow, namely, resistance, pressure coefficient, and cavitation number, were reduced to the conditions at infinity using extrapolation curves as a function of the relative constriction of the flow to zero value of the constiction. It was shown that use of an equivalent velocity is not always justified and may lead to errors. The cause of these errors resides in the fact that the increase in equivalent velocity at the profile of the model is nonuniform and is affected not only by the flow boundaries but also by the stage of cavitation, i.e., the shape and size of the cavitation zone. The true value of the correction can be determined only be a series of experiments at different flow constrictions but the same Reynolds number.

NOTATION

a height of working chamber; b width of working chamber; b_c maximum width of cavitation zone; C_x drag coefficient; d diameter of cylinder; h its height; g acceleration of gravity; k equivalent velocity coefficient; l_c length of cavitation zone; l_0 length of separated part of zone; N number of separating cavities; p hydromechanical pressure; p_c pressure in cavitation zone; p_v water vapor pressure; P pressure coefficient; q velocity head; r radius of cylinders; R Reynolds number; S Strouhal number; u velocity at cylinder profile; v flow velocity upstream from model; v_∞ flow velocity with allowance for constiction due to model—equivalent velocity; X drag of model; β relative width of cavitation zone; Δv velocity increment; γ weight of unit volume of water; $\varkappa$ cavitation number; λ_c relative length of cavitation zone: λ_0 relative length of separated part of cavitation zone; ν kinematic viscosity; θ angle between axis of piezo aperture in model and direction of flow.

Subscripts: ∞—for flow of infinite width; b—for flow of finite width:

$$\varkappa = \frac{p_\infty - p_v}{\gamma q}, \qquad P = \frac{p - p_\infty}{\gamma q}, \qquad q = \frac{v^2}{2g}, \quad R = \frac{dv_\infty}{\nu},$$

$$S = \frac{Nd}{v_\infty}, \quad C_x = \frac{X}{hdq_\infty \gamma}, \quad k = \frac{b}{b-d}, \quad \lambda_c = \frac{l_c}{d},$$

$$\lambda_0 = \frac{l_0}{d}, \quad \beta = \frac{b_c}{d}. \tag{1}$$

1. Owing to the presence of the walls opposite the end faces of the model, the uniform velocity curve at the measuring section is distorted by the boundary layer as a result of which the drag distribution along the model also becomes nonuniform. The effect of these walls can be eliminated, if the drag is determined from the pressure distribution at the mid-section of the model and the measured pressure values are related to the velocity at the axis of the working chamber.

However, the presence of the other two walls must also have an effect on the flow past the middle element of the model as compared with a flow of infinite width.

There have been more theoretical than experimental studies of the effect of flow boundaries [1-15]. The theoretical studies relate primarily to the influence of chamber width on drag, lift force, and the dimensions of the cavitation zone, generally for a very small range of the ratio d/b and small cavitation numbers $\varkappa$.

It has been recommended that the measured values of the hydrodynamic pressures be referred to a velocity equivalent to the velocity of a flow of infinite width and computed, in our investigations, from the formula [16-18]

$$v_\infty = vk = v\frac{b}{b-d}. \tag{2}$$

In studies of cavitation erosion [19, 20] it is often necessary to know the effect of the boundaries over a quite wide range of values of the ratio d/b on other characteristics of the flow, for example: the Reynolds number R, the Strouhal number S, and the cavitation number $\varkappa$ for different stages of cavitation λ.

Table 1

d, mm	d/b	v, msec^{-1}	k	$R\cdot 10^{-5}$
5	0.05	12	1.03	0.62
8	0.16	7.5	1.09	0.65
10a	0.20	12.5	1.1	1.38
10b	0.20	9.2	1.1	1.02
12	0.24	10	1.18	1.46
15	0.30	4.4	1.21	0.80
20	0.40	5.86	1.44	1.68

2. The experiments were conducted in a water tunnel, cross section of working chamber $a \times b$ = 20 × 50 mm. Six cylindrical models were tested at ratios d/d = 0.1 - 0.4. The test conditions are shown in Table 1. The Reynolds numbers R in Table 1 were calculated using values of k obtained as a result of the present investigation with extrapolation of C_x. As Table 1 shows, in these experiments the influence of the Reynolds number on the drag C_x was relatively small.

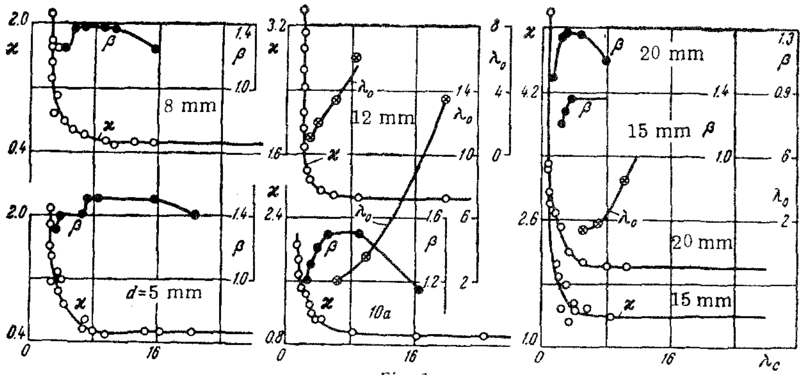

Fig. 1

The experiments consisted of: 1) observations of the development of a cavitation zone in relation to the cavitation number, accompanied by measurements of the dimensions of the cavitation zone; 2) measurements of the pressure distribution at the mid-section of the models, the water temperature, and the barometric pressure. In order to measure the pressure at different points of the cross section, using the same piezo aperture, the model, connected with a vernier scale, could be adjusted at an angle θ between the flow axis and the axis of the piezo aperture. The flow axis was found as the axis of symmetry of the pressure coefficient curves $P = P(\theta)$.

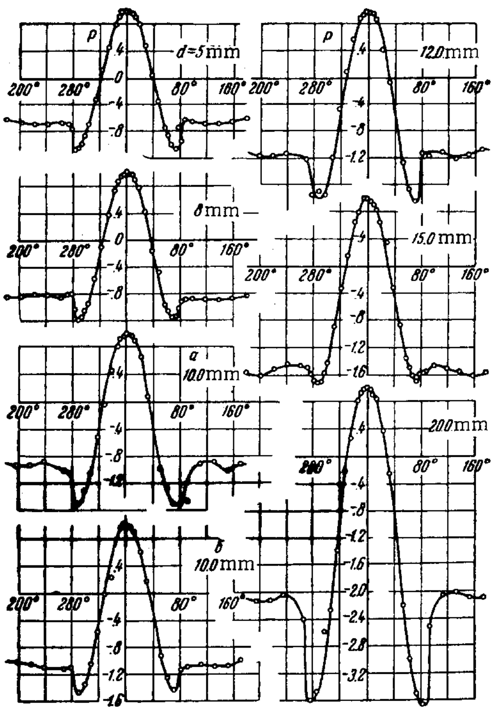

Fig. 2

The drag of the cylinders was found by planimetry from the pressure graphs $P = f(r)$ without allowance for friction drag, which forms only a small fraction of the total drag (1-2%).

The results of the experiments were analyzed using extrapolation curves, on which the value of the parameter was plotted as a function of the ratio d/b. The value of the parameter obtained by extrapolation to the value d/b = 0 was taken as equal to that for a flow of infinite width.

3. The results of visual measurements of the cavitation zone are presented in the graphs showing the lengths $\lambda_c = l_c/d$ and $\lambda_0 = l_0/d$ and the maximum width of the zone $\beta_c = b_c/d$ as a function of the ratio d/b (Fig. 1); the graph for d = 10 mm was constructed for $R = 1.38 \cdot 10^5$.

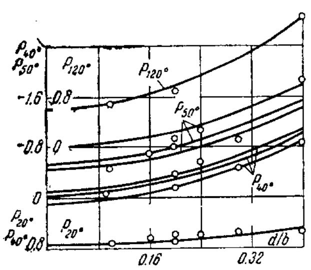

Fig. 3

Some idea of the structure of the zone is given by λ_0, the length of the separated part of the zone. The length l_c is defined as the distance from the axis of the model to the point at which the jets bounding the cavitation zone join, while the length l_0 is defined as the distance from the axis of the model to the point of transition from the part of the cavity adjacent to the model to the part with a bubbly structure at the end of the cavity. As found previously [21, 22], the pressure in the hollow part is equal to the water vapor pressure p_v in the bubbly part of the cavity p_c/p_v. An examination of Fig. 1 leads to the following conclusion.

In spite of the similarity of the Reynolds numbers for the models in each of the two groups (diameters 5, 8, and 15 mm and diameters 10, 12, and 20 mm), there is no similarity in the development of the cavitation zones. In all stages, with increase in the ratio d/b the cavitation zone develops closer to the model as a result of an increase in positive pressure gradient [23]. Thus, for models 5 and 8 mm in diameter in the initial stage we have $\lambda_c = 3$ and for a model 15 mm in diameter $\lambda_c = 1$. At the same time, with increase of d/b the maximum width of the cavitation zone moves closer to the model and decreases from $\beta = 1.5$ to $\beta = 1.25$. For the separated stage, at $\lambda = \infty$, the number $\varkappa = 0.5$, whereas for zero pressure gradient it must be equal to 0.

The pressure distribution of six cylinders in the absence of cavitation is shown in Fig. 2, the pressure coefficient relation P(d/b) in Figs. 3, 4, the drag of these cylinders $C_x(d/b)$ and the angles corresponding to the characteristic points on the pressure distribution curves $\theta_{min}(d/b)$, $\theta_{180°}(d/b)$, in Fig. 5. Applying the extrapolation method to $C_x(d/b)$, we can express it by means of the two formulas

$$C_x = 0.81 + 5(d/b)^2, \qquad C_x = 0.74 + 5(d/b)^2. \tag{3}$$

These differ with respect to the constant, possibly as a result of the difference in Reynolds numbers (Table 1), except for the C_x for the cylinder d = 15 mm. The drag of this cylinder is considerably different from that which might have been expected from (3). This discrepancy may be attributed to the fact that the pressure P_{min} is less than what it should be according to the relation $P_{min}(d/b)$ for the cylinders with d = 5, 8 and 10 mm (Fig. 4). In its turn, the deviation of P_{min} from the general law is probably due to the unique character of the dependence of the angle $\theta_{min}(d/b)$. There is a certain value $d/b \approx 0.20$, at which θ_{min} assumes a minimal value and after which the point with the value P_{min} begins to move in the direction of flow (Fig. 5).

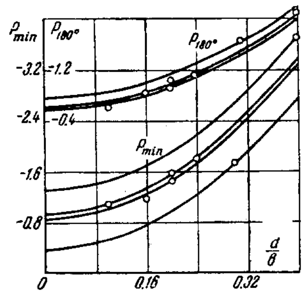

Fig. 4

Accepting this explanation of the deviation of C_x and P_{min} for d = 15 mm from the general laws in terms of the simultaneous opposing influence of the flow boundaries and the Reynolds number, in the subsequent analysis of the pressure distribution over the cylinders we used the extrapolation method (Figs. 3, 4).

Employing this method, we shall consider the question of the value of the equivalent velocity for reducing the pressure to the conditions for a flow of infinite width. In accrodance with the definition of the pressure coefficient P (1), the increase in velocity at different points of the mid-section of cylinders in a flow of infinite width and a flow of finite width may be found from the formulas

$$\Delta v_\infty = \frac{u - v_\infty}{v_\infty} = \sqrt{1 - P_\infty} - 1, \qquad \Delta v_b = \frac{u_b - v_\infty}{v_\infty} = \sqrt{1 - P_b} - 1, \tag{4}$$

where the subscript ∞ indicates that the given quantity relates to a model with d/b = 0, and the subscript b to a model with d/b > 0.

Then the influence of the flow boundaries on the velocity at any point of the mid-section may be estimated from the velocity increment in accordance with the formula

$$\Delta v = \frac{\Delta v_b - \Delta v_\infty}{v_\infty}. \qquad (5)$$

The results of computations of Δv are presented in the graph of $\Delta v(d/b)$ for six cylinders, two curves being given for the cylinder d = = 10 mm: at velocities v = 9.2 and 12.5 msec^{-1} (Fig. 6). The increase

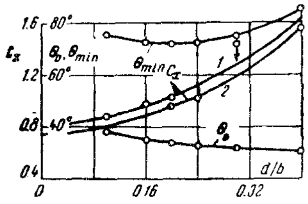

Fig. 5

in equivalent velocity over the profile is nonuniform. The changes in Δv are most considerable in the leading half of the profile and at large ratios d/b. This means that to correct the pressure coefficient for the effect of the flow boundaries the equivalent velocity must be found for each point of the profile. It is not permissible to use formula (2), although for such an accurate consideration of the effect of flow width laborious experiments are required. To avoid these difficulties in finding C_x by the pressure distribution method, it is necessary find C_x from the unreduced pressures, determine the equivalent velocity v_∞, using the extrapolation of C_x to the value d/b = 0, and find the equivalent velocity increment from the formula

$$\Delta v = \left(\frac{C_{xb}}{C_{x_\infty}}\right)^{1/2} - 1. \qquad (6)$$

The results of computing Δv from the data of our experiments and from the data of other authors are presented in Fig. 7, in which: 1 denotes experimental data obtained in our investigations, 2 the same according to Fage [2], and 3 the same according to Thoma [1]; the broken line represents the theoretical relation of [3], the solid curve was constructed using points 1. The relation $\Delta v(d/b)$ is satisfactorily described by the equation of a parabola

$$\Delta v = 300\,(d/b)^2. \qquad (7)$$

The experimental values for Δv obtained by all the authors differ considerably from the theoretical values.

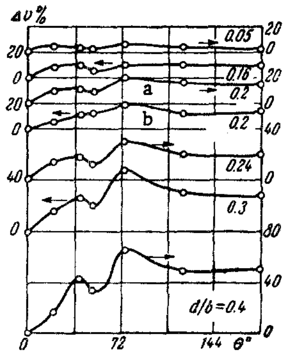

Fig. 6

To reduce the values of the cavitation number to the conditions corresponding to an infinitely wide flow, it is necessary to use the same method as for C_x. These relations have been plotted on the graph of $\varkappa$ (d/b) for three stages of cavitation: for the initial stage $\lambda_c = 0$, for

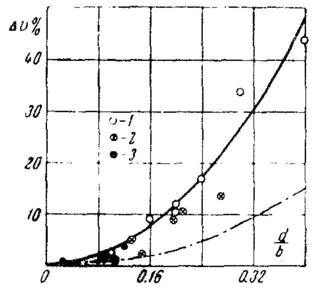

Fig. 7

the stage $\lambda_c = 3$, and for the separated stage $\lambda_c = \infty$ (Fig. 8). The increments Δv, determined from this graph, depend on the stage of cavitation. For each state of cavitation there is a particular equivalent velocity for reducing $\varkappa$ to the conditions corresponding to an infinite flow. For cylinders d = 10 and 12 mm the least value of Δv is that for $\lambda_c = 3$. The values of Δv increase both in the direction of $\lambda_c = 0$ and in the direction of $\lambda = \infty$.

The results of the experiments to elucidate the effect of the boundaries on C_x and P can also be used to clarify the question of the value of the equivalent velocity in computing the Strouhal number S.

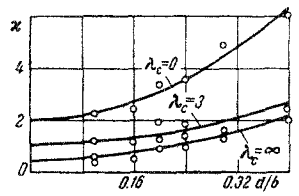

Fig. 8

For developed cavitation it is possible to write for the jet bordering the cavity, from the section M to the section of maximum constriction of the jet by the cavity M_c,

$$\frac{p_\infty}{\gamma} + \frac{v^2}{2g} = \frac{p_c}{\gamma} + \frac{v_c^2}{2g}. \qquad (8)$$

From Eq. (8) we find the pressure coefficient at the point M_c

$$P_c = \frac{p_c - p_\infty}{\gamma q} = 1 - \frac{v_c^2}{v^2} \qquad (9)$$

analogous to the pressure coefficient at some point of the model (see formula for P (1)).

We can put $p_c = p_v$, and then

$$\varkappa = -P_c. \qquad (10)$$

At constant velocity v, the cavitation number will be a function of the velocity v_c. The number of separating cavities $N = Sv_c/d$ must also be found in relation to these changes in v_c. On this basis, for example, in experiments to determine S for models with d = 10 and 12 mm at $\lambda = 3$ the equivalent velocity v may be equal to the velocity obtained from the extrapolation curve for $\varkappa$ in the above-mentioned stages of cavitation. To reduce the Reynolds number to the conditions for an infinite flow, it is logical to take the increments Δv equal to the velocity increments for computing C_x.

By way of example, Table 2 gives the results of determining by the extrapolation method the values of Δv (%) which must be used to reduce the values of the parameters (first column) to the conditions at infinity for flow past cylinders with d = 10 and 12 mm.

Table 2

Parameter	d, mm		Parameter	d, mm	
	10	12		10	12
C_x	12	17.5	$\varkappa_{\lambda=0}$	24	33
R	12	17.5	$\varkappa_{\lambda=2-2.5}$	18	26
$P_{50°}$	19	22	$\varkappa_{\lambda=\infty}$	39	52
P_{min}	17	30	S	18	26
$P_{180°}$	28	41	acc. to (2)	25	32

The last row of Table 2 gives Δv computed from formula (2).

CONCLUSIONS

1. Reduction of the experimental data characterizing cavitating flow past a cylinder in the straight working chamber of a water tunnel to the conditions corresponding to a flow of infinite width using the equivalent velocity found from the continuity equation may lead to errors in determining these parameters.

2. The cause of these errors resides in the fact that the increase in equivalent velocity over the profile of the cylindrical model is nonuniform and is affected not only by the flow boundaries but also by the stage of development of cavitation, i.e., the change in the shape and dimensions of the cavitation zone as a function of the cavitation number. The true value of the correction can be found only from a series of experiments at different ratios d/b for identical Reynolds numbers and cavitation stages using the extrapolation method.

3. According to studies of the scale effect by the method described in [20], in experiments on cavitation erosion it is necessary to use the cavitation drag for cylinders determined under identical conditions of constriction of the working chamber by the model. In comparing the results of experiments conducted in different chambers it is necessary take into account the effect of a possible difference in pressure gradients along the axis of the chambers on the value of the cavitation parameters.

REFERENCES

1. A. Thom "An Investigation of fluid flow in two dimensions," Rept and Memoranda, no. 1194, 1929.

2. A. Fage, "On the two-dimensional flow past a body of symmetrical cross-section mounted in a channel of finite breadth," Rept and Memoranda, no. 1223, 1929.

3. A. Toussaint and E. Jacobs, "Methods of aerodynamic research," in: Aerodynamic Theory (ed. Durand) [Russian translation], vol. III, Gostekhizdat, 1939.

4. M. Kinoshita and J. Shinoda, "Über den Wandeinfluss auf das Flugelprofil mit einer vollig ausgebildeter Kavitation," Meeting Soc. Naval Archit. Japan Special Paper, 1944.

5. M. I. Gurevich, "Symmetrical cavitating flow past a flat plate," Izv. AN SSSR, OTN, no. 4, 1946.

6. M. I. Gurevich, "Drag of a cylinder and a wedge at low cavitation numbers," Tr. Mosk. in-ta rybn. promyshl. i khoz., no. 5, 1953.

7. G. Birkhoff, M. Plesset, and N. Simons, "Wall effects in cavity flow," Quart. Appl. Mech., P. I, T. 8, no. 2, 1950; 9, no. 4, p. 11, 1952.

8. H. Cohen, C. D. Sutherland, and O. Tu Yih" Wall effects in cavitating hydrofoil flow," J. Ship. Res., vol. 1, no. 3, 1957.

9. H. Cohen and R. Gilbert "Two-dimensional study cavity flow about slender bodies in channels of finite breadth," J. Appl. Mech. Trans ASME, vol. 79, pp. 170-176, 1957.

10. R. L. Waid "Water tunnel investigations of two-dimensional cavities." Hydrodynamics Laboratory Report Cal. Inst. of Techn., no. E-73.6, 1957.

11. B. R. Parkin "Experiments on circular ARC and Flat Plate Hydrofoils," J. Ship Res. vol. 1, no. 4, 1958.

12. H. Cohen and O. Tu Yih, "A comparison of wall effects on supercavitating flows past symmetric bodies in solid wall channels and jet" Proc. Ninth Intern. Congr. Appl. Mech., Brussels, vol. 2, pp. 359–370, 1950.

13. I. J. Campbell and G. E. Thomas "Water tunnel boundary effects on axially symmetric fully developed cavities," Admiralty Res. Lab. (RI) GH, 1 (18), 1956.

14. A. H. Armstrong and K. G. Tadman, " Wall corrections to axially symmetric cavities in circular tunnels and jets," Ministry of Supply A. R. E., Report no. 7/52, 1953.

15. H. Cohen and R. C. Di Prima, " Wall effects in cavitating flows," Second Symposium Naval Hydrodynamics, 1958.

16. G. Birkhoff, Hydrodynamics [Russian translation], IL, 1954.

17. V. A. Konstantinov, "Effect of Reynolds number on separated flow," Izv. AN SSSR, OTN, no. 10, 1964.

18. S. N. Nasilov and B. M. Teverovskii, "Cavitating flow past a cylinder," Tr. VIGM, vol. 29, 1961.

19. K. K. Shal'nev, "Energy parameter and scale effect in cavitation erosion, "Izv. AN SSSR, OTN, no. 5, 1961.

20. I. I. Varga, B. A. Chernyavskii, K. K. Shal'nev, "Method of investigating the scale effect in cavitation erosion," PMTF, no. 3, 1963.

21. K. K. Shal'nev, " Shape of cavitation region in a normal flow past a plate," DAN, vol. 94, no. 6, 1954.

22. K. K. Shal'nev, "Pressure and erosion in the region of separated caviation for a circular profile," Izv. AN SSSR, OTN, no. 6, 1954.

23. K. K. Shal'nev, "Effect of pressure gradient in the flow on the development of a cavitation zone," PMTF, no. 1, 1961.

5 September 1964 Moscow